Lecture 5

Soil Water Characteristic Curves (SWCC)
Surface Tension, Review
The capillary model provides a mathematical relationship between the radius of curvature of the air-water meniscus and the pressure difference between the air and water phases.

The difference between the air pressure $u_a$ and the water pressure $u_w$ was shown to be balanced by the surface tension $T_s$ acting at the wetting angle $\alpha$ along the solid surface.
Matric Suction and Soil Strength

- Surface tension has the ability to support a column of water in a capillary tube.

- The surface tension associated with the contractile skin places a reaction force on the wall of the capillary tube.

- The vertical component of this reaction force produces compressive stresses on the wall of the tube. In other words, the weight of the water column is transferred to the tube through the contractile skin.

- The contractile skin results in an increased compression on the soil structure in the capillary zone.

- Therefore the presence of matric suction in an unsaturated soil produces a volume decrease and an increase in the shear strength of the soil.
Soil-Water Potential

- **Definition**: The amount of work that must be done per unit of a specified quantity of pure water in order to transport reversibly and isothermally an infinitesimal quantity of water from a specified source to a specified destination.

- Total Soil-water potential can be categorized into four subgroups as:
  - **Matric Potential** ($\psi_m$)
    - As we discussed in previous lecture, this component is related to the difference between pore air and pore water pressure in soils. $\psi_m = U_a - U_w$
  - **Osmotic Potential** ($\psi_o$)
    - This component is related to the salt content in the water. $\psi_o = \nu CRT_k$
  - **Pressure Potential** ($\psi_p$)
    - Recall from your CE4348, the pressure potential can be calculated as: $\psi_p = \rho_w gh$
  - **Gravitational Potential** ($\psi_g$)
    - Is related to the weight of the materials and can be calculated as: $\psi_g = MgZ$
Stage a: Drainage of meniscus water from inter-aggregate or inter-particle pores

Stage b: Drainage of bulk water from inter-aggregate or inter-particle pores

Stage c: Drainage of meniscus water from inter-aggregate or inter-particle pores

Stage d: Drainage of bulk and meniscus water from intra-aggregate pores
Total, Matric, and Osmotic Suctions for Glacial Till (Krahn and Fredlund, 1972)
Components of Soil Suction and Total Suction for Regina Clay (From Fredlund, 2002).
Designation of the Amount of Water in Soils

The amount of water in the soil can be defined using more than one variable. Variables used to define the amount of water in the soil are:

i. Gravimetric water content \( w \),

\[
w = \frac{M_w}{M_s}
\]

ii. Volumetric water content \( \theta \),

\[
\theta = \frac{V_w}{V_v + V_s}
\]

iii. Degree of saturation \( S \),

\[
S = \frac{V_w}{V_v}
\]

iv. Volume of water, \( V_w \), referenced to the original volume of the specimen, \( V_0 \) (i.e., \( V_w/V_0 \)).
Gravimetric vs. Volumetric Water Content Example

During field trial, the field engineer collected a 200 cm$^3$ soil sample. Its moist weight was measured as 150 g. After drying, the dry weight was 100 g. Determine the volumetric and gravimetric water contents of this soil.

- **Gravimetric Water Content:**

$$w = \frac{\text{Moist Weight} - \text{Dry Weight}}{\text{Dry Weight}} = \frac{150 - 100}{100} = 50\%$$

- **Volumetric Water Content:**

$$\theta = \frac{\text{Volume of Water}}{\text{Total Volume}} = \frac{150 - 100}{200} = 25\%$$
## Advantages and Disadvantages of Various Designations for Amount of Water in Soil

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<thead>
<tr>
<th>Designation</th>
<th>Advantages</th>
<th>Disadvantages</th>
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| Gravimetric water content, $w$ | - Consistent with usage in classic soil mechanics  
- Most common means of measurement  
- Does not require a volume measurement  
- Reference value is a “mass of soil” which remains constant | - Does not allow differentiation between change in volume and change in degree of saturation  
- Does not yield the correct air-entry value when the soil changes volume upon drying |
| Volumetric water content, $\theta$ | - Is the basic form that emerges in the derivation of transient seepage and fluid storage in unsaturated soils  
- Commonly used in databases of results obtained in soil science and agronomy | - Requires a volume measurement  
- Rigorous definition requires a volume measurement at each soil suction  
- Is the designation least familiar and least used historically in geotechnical engineering |
| Degree of saturation, $S$     | - Most clearly defines the air-entry value  
- Appears to be the variable most closely controlling unsaturated soil property functions | - Requires a volume measurement  
- Although volume measurements are required, the degree of saturation variable does not quantify overall volume change |
The Soil Water Characteristic Curve (SWCC)

- The Soil Water Characteristic Curve (SWCC) describes the functional relationships between soil water content (θ, w, or Sr) and matric potential under equilibrium conditions.
- The SWCC is an important soil property related to pore space distribution (sizes, inter-connectedness), which is strongly affected by texture, void structure and other factors such as organic content.
- The SWCC is a primary hydraulic property required for modeling water flow in porous materials.
- The soil-water function is highly nonlinear and relatively difficult to obtain accurately.
The Soil Water Characteristic Curve

Early conceptual models for SWC curve were based on the "bundle of cylindrical capillaries" (BCC) representation of pore space geometry (Millington and Quirk, 1961).

The BCC representation postulates that at a given matric potential a portion of interconnected cylindrical pores are completely filled with liquid, whereas larger pores are completely empty.
The Soil Water Characteristic Curve- Bundle of Cylindrical Capillaries (BCC) Analogy

This convenient idealization of soil pore space, enables a linkage between the soil pore size distribution and the SWCC based on capillary rise equation:

$$h_i = \frac{2T \cos \alpha}{\rho_w g r_i}$$
Soil Water Characteristic Curves Also Represent Water Content as a Function of Height Above Water Table
Effect of Particle Size on the Height of the Capillary Rise
Relationship Between Matric Suction and Pore Size for Various Soils

Surface tension, $T_s = 72.75$ mN/m at $t = 20^\circ$C

Matric suction, $\mu_a - \mu_w$ (kPa)

Pore radius, $r$ (mm)
Relationship Between Void Ratio and Soil Suction
There is a strong linkage between the SWCC and the unsaturated soil properties and this relationship must be maintained throughout the numerical modeling process.
Typical SWCC for Soils of Different Texture
Matric Suction and Fines Content

\[ P_{200} PI = P_{200} \times PI \]
Drying Phenomenon Showing Relationship Between Air-Entry Value (AEV) and Shrinkage Limit of a Soil (After Marinho, 1994).

- Drying of a saturated soil (e.g., starting at point A) follows the saturation line until air begins to enter the largest soil voids at point B.

- Point B is an indication of the air-entry value (AEV) of an initially slurry soil.

- As the soil continues to dry, it reaches a minimum void ratio beyond which there is no further volume change. When the water content of the soil reaches zero, the soil suction has increased to 1,000,000 kPa (i.e., point D) which is the upper limit of the soil suction.
Shrinkage Curve
Air Entry Value (AEV) and Residual water Content ($\theta_r$)

- **The air-entry value** of the soil is the matric suction where air starts to displace water in the largest pores in the soil.

- **The residual water content** is the water content where a larger suction change is required to remove additional water from the soil. In other words, there is a change in the rate at which water can be extracted from the soil.
Identification of the Zones on the SWCC
Hysteresis and Scanning Curves

- Tension of the soil for a given water content is not unique, but depends on the soils history of wetting and drying, therefore the SWCC is not a unique curve.

- During wetting the small pores fill first and during draining the large pores empty first.

- The intermediate loops in the plot are called the scanning curves, indicating the transitions between the wetting and drying branches.

- Meniscus radii is greater in an advancing fluid than in a retreating one.

- Entrapped air in a newly wetted soil decreases water content per unit suction.

- Clay-rich soils change geometry through swelling and shrinking during wetting and drying.
Ink Bottle Analogy during Sorption and Desorption

Visualization of the ink bottle analogy for equilibrium height of water in a variable width pore:

- **Capillary Drainage** (Desorption or Drying)
- **Capillary Rise** (Sorption or Wetting)

During wetting the small pores fill first and during draining the large pores empty first.
Influence of Height, Radius, and Shape Effects on Capillary Action
\[ \gamma = \frac{F}{L \cos \theta} \]

Where:
- \( \gamma \) : Surface tension
- \( F \) : Measuring force (force acting on the plate)
- \( L \) : Perimeter of plate
- \( \theta \) : Contact angle of plate and the liquid
Hysteresis of SWCC, Cont.

- The drying and wetting SWCCs are significantly different, and in many cases it becomes necessary to differentiate the soil properties associated with the drying curve from those associated with the wetting curve.

- This means that the geotechnical engineer must make a decision regarding the process that is to be simulated.

- It might also be appropriate in some cases to use an average SWCC (i.e., between the drying and wetting SWCCs) when estimating unsaturated soil property functions.
Heaved walk and step
Flat if telepost adjusted correctly
Heaved floor
Heaved lawn
Active zone
FOUNDATION MOVEMENT DUE TO SOIL MOISTURE CONTENT CHANGES

Diagram 1

- Seeping Tile
- 4" Concrete Floor Slab
- Telepost
- HEAVY due to swelling
- SETTLEMENT due to tree

30"x10" Typical Concrete Footing

Image of a cracked floor with a potential cause related to soil moisture content changes.
Snow pile

Pavement

Frost heaving

Longitudinal crack

Snow pile

FROZEN ZONE

Ice lenses

Unfrozen frost susceptible natural soil

Sinking frost line

Water pumping feeding the ice lenses
Frost Heaving

- Soils containing water that expands when frozen, moving the soil upward.

Large scale palsa formed by winter frost heave, then cut by summer meltwater.
SWCC Models
Fredlund and Xing (1994)
Best-fit Curves Fitted To Experimental Data Using Three Representations Of Water Content

- Gravimetric water content (experimental data)
- Volumetric water content (experimental data)
- Degree of saturation (experimental data)

Void ratio: 0.756
Initial water content: 43.5%
Total density: 2.60 mg/m³
Fredlund and Xing proposed the following relationship between volumetric water content and soil suction:

$$\theta = \theta_s \left[ \frac{1}{\ln[e + (\psi/a_f)^{n_f}]} \right]^{m_f}$$

- $a_f$, $n_f$, $m_f$ = Three fitting parameters,
- $\theta$ = Volumetric water content
- $\theta_s$ = Saturated volumetric water content, and
- $\epsilon$ = Irrational constant equal to 2.71828.
- $\psi$ = Soil suction
Sensitivity Analysis of Shape Parameters of the Fredlund-Xing Equation

\[ \theta = \theta_s \left[ \frac{1}{\ln(e + (\psi/a_f)^{m_f})} \right]^{m_f} \]

Effect of \( a_f \) soil parameter on SWCC when \( n_f = 2 \) and \( m_f = 1 \).
Sensitivity Analysis of Shape Parameters of the Fredlund-Xing Equation

\[ \theta = \theta_s \left[ \frac{1}{\ln\left[ \epsilon + \left( \frac{\psi}{\alpha_f} \right)^{n_f} \right]} \right]^{m_f} \]

Effect of \( m_f \) soil parameter on SWCC when \( n_f = 2 \) and \( \alpha_f = 100 \)
Variation of Soil Suction with Particle Size and Water Content
Sensitivity Analysis of Shape Parameters of the Fredlund-Xing Equation

$$\theta = \theta_s \left[ \frac{1}{\ln(e + (\psi/a_f)^{m_f})} \right]^{n_f}$$

Effect of $n_f$ soil parameter on SWCC when $m_f = 1$ and $a_f = 100$
Zapata (1999) used a database of approximately 190 soils collected from previously published data.

- The soils collected were divided into two categories: plastic and non-plastic soils.
- The database consisted of approximately 70 plastic soils and 120 non-plastic soils.
- The data collected for the plastic soils consisted of the percentage of passing the No. 200 sieve and the Atterberg limits, in particular, the plasticity index.
- The grain-size diameter $D_{60}$ was used to represent non-plastic soils.
- Each soil used in the statistical correlation had a measured and well-defined SWCC.
- The weighted value of PI, which is the percentage passing the No. 200 sieve (used as a decimal value) multiplied by the plasticity index (i.e., $w_pPI$) was used to characterize the plastic soils.

- The correlation study yielded a family of SWCCs for both plastic and non-plastic soils. The results of the Zapata’s study have been used as part of the Enhanced Integrated Climatic Model (EICM) in the new Mechanistic Empirical Pavement Design Guide (MEPDG).
Family of SWCCs for Non-Plastic Soils Based on Zapata’s Model (1999)

\[ \theta = \theta_s \left[ \frac{1}{\ln[e + (\psi/a_f)^{nf}]} \right]^{mf} \]

\[ a_f = 0.8627(D_{60})^{-0.751} \]

\[ m_f = 0.1772[\ln(D_{60})] + 0.7734 \]

\[ n_f = 7.5 \]

\[ h_r = a_f \left( \frac{1}{D_{60} + 9.7e^{-4}} \right) \]
Family of SWCCs for Plastic Soils Based on Zapata’s Model (1999)

\[\theta = \theta_s \left[ \frac{1}{\ln[e + (\psi/a_f)^{n_f}]} \right]^{m_f}\]

\[a_f = 0.00364(w_{PI})^{3.35} + 4(w_{PI}) + 11\]

\[m_f = 0.0514(w_{PI})^{0.465} + 0.5\]

\[n_f = m_f (-2.313(w_{PI})^{0.14} + 5)\]

\[h_r = a_f (32.44e^{0.0186(w_{PI})})\]
Combined Family of SWCCs for Plastic and Non-plastic Soils (after Zapata, 1999).
Estimation of the Model Parameters for Granular Soils - Torres, 2011

Torres (2011) analyzed a database of about 4500 granular soils and found that the $D_{10}$ particle size produced improved correlation coefficients to those obtained when using the $D_{60}$ particle size. The correlation coefficients were obtained for the Fredlund and Xing (1994) SWCC equation.

\[ a_f = -967.2D_{10}^2 + 218.4D_{10} - 2.70 \]

\[ \log n_f = -0.0075a_f^3 + 0.1133a_f^2 - 0.3577a_f + 0.3061 \]

\[ m_f = 0.0058a_f^3 - 0.0933a_f^2 + 0.4069a_f + 0.3481 \]

The soil suction at residual conditions, $\Psi_r$, was found to be relatively constant for all granular soils:

\[ \Psi_r = 100 \text{ kPa} \]
Application of SWCC

- The SWCC constitutes the primary soil information required for the analysis of seepage, shear strength, volume change, air flow, and heat flow problems involving unsaturated soils (Fredlund, 2000).

- The permeability function and the water storage function for an unsaturated soil are related to the SWCC.

- The coefficient of permeability is known to become extremely low and hydraulic flow would appear to essentially cease near residual conditions, giving way to vapor flow. In other words, there appears to be a change from liquid flow to vapor flow as residual suction is exceeded.

- The effective angle of internal friction can be applied to a soil up to the air-entry value. The friction angle then decreases until it becomes essentially zero at soil suctions exceeding residual soil suction. Recent research on a number of soil types indicate that the angle of friction tends toward zero at high soil suctions (Fredlund, 2000).
# Devices for Measuring Soil Suction

<table>
<thead>
<tr>
<th>Name of Device</th>
<th>Suction Component Measured</th>
<th>Range, kPa</th>
<th>Comments</th>
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</thead>
<tbody>
<tr>
<td>Psychrometers (Peltier type)</td>
<td>Total</td>
<td>100(^a) to (\sim)8000</td>
<td>Constant-temperature environment required</td>
</tr>
<tr>
<td>Filter paper</td>
<td>Total</td>
<td>Entire range</td>
<td>May measure matric suction when in good contact with moist soil</td>
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<tr>
<td>Tensiometers</td>
<td>Negative pore-water pressures or matric suction when pore-air pressure is atmospheric</td>
<td>0–90</td>
<td>Difficulties with cavitation and air diffusion through ceramic cup</td>
</tr>
<tr>
<td>Null-type pressure plate (axis translation)</td>
<td>Matric</td>
<td>0–1500</td>
<td>Range of measurement is a function of the air-entry value of ceramic disk</td>
</tr>
<tr>
<td>Thermal conductivity sensors</td>
<td>Matric</td>
<td>10 to (\sim)1500</td>
<td>Indirect measurement using variable-pore-size ceramic sensor</td>
</tr>
<tr>
<td>Pore fluid squeezer</td>
<td>Osmotic</td>
<td>Entire range</td>
<td>Used in conjunction with psychrometer or electrical conductivity measurement</td>
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\(^a\)Controlled temperature environment to \(\pm 0.001^\circ\)C.
Range of Suction Values Measured with Various Tests
Relationship between Volume-Mass Properties of Soils

![Graph showing the relationship between volume, mass, and properties of soils.](chart.png)